AD

SUPPLEMENT
TO
INVERSE SOLUTION OF LONG GEODESICS

by Emanuel M. Sodano

June 1966

US ARMY ENGINEER
GEODESY, INTELLIGENCE AND MAPPING RESEARCH AND DEVELOPMENT AGENCY
FORT BELVOIR, VIRGINIA

U. S. ARMY ENGINEER
GEODESY, INTELLIGENCE AND MAPPING
RESEARCH AND DEVELOPMENT AGENCY
FORT BELVOIR, VIRGINIA

SUPPLEMENT
TO
INVERSE SOLUTION OF LONG GEODESICS

For presentation
at
Second International Symposium
on
Geodetic Calculations
Brussels, Belgium
6 - 11 June 1966

by
Emanuel M. Sodano
Research Institute for Geodetic Sciences

ABSTRACT

Alternate formulas are given for the rigorous non-iterative solution
of very long (as well as medium and very short) geodesics. They are not
only shorter and simpler than the author's original version published in
the Bulletin Geodesique, but the powers of the spheroid parameter can be
factored out in the same manner as in the corresponding solution of the
Direct Geodetic Problem. Theoretical, as well as practical, significance
is noted.

SUPPLEMENT
TO
INVERSE SOLUTION OF LONG GEODESICS

by
Emanuel M. Sodano

At the bottom of page 75 of [1], the following main power series formula is given for the calculation of the Direct Geodetic Problem for long lines:

$$\phi_0 = [\phi_S]$$

$$+ a_1 \ [-\frac{e'^2}{2} \sin \phi_S]$$

$$+ m_1 \ [-\frac{e'^2}{4} \phi_S + \frac{e'^2}{4} \sin \phi_S \cos \phi_S]$$

$$+ a_1^2 \ [\frac{5e'^4}{8} \sin \phi_S \cos \phi_S]$$

$$+ m_1^2 \ [\frac{11e'^4}{64} \phi_S - \frac{13e'^4}{64} \sin \phi_S \cos \phi_S - \frac{e'^4}{8} \phi_S \cos^2\phi_S$$

$$+ \frac{5e'^4}{32} \sin \phi_S \cos^3\phi_S]$$

$$+ a_1 m_1 [\frac{3e'^4}{8} \sin \phi_S + \frac{e'^4}{4} \phi_S \cos \phi_S - \frac{5e'^4}{8} \sin \phi_S \cos^2\phi_S] \ \text{radians}$$

At the bottom of page 73 of [1] (derived by change of parameters from the bottom of page 19 of [2]), the following main power series formula is given for the calculation of the Inverse Geodetic Problem for long lines:

/

$$\frac{S}{b_0} = \quad [(1 + f + f^2)\phi]$$

$$+ \ a \ [(f + f^2) \sin \phi - (\frac{f^2}{2})\phi^2 \csc \phi]$$

$$+ \ m \ [- (\frac{f + f^2}{2})\phi - (\frac{f + f^2}{2}) \sin \phi \cos \phi + (\frac{f^2}{2})\phi^2 \cot \phi]$$

$$+ \ a^2[- (\frac{f^2}{2}) \sin \phi \cos \phi]$$

$$+ \ m^2[(\frac{f^2}{16})\phi + (\frac{f^2}{16}) \sin \phi \cos \phi - (\frac{f^2}{2})\phi^2 \cot \phi - (\frac{f^2}{8}) \sin \phi \cos^3\phi]$$

$$+ \ am[(\frac{f^2}{2})\phi^2 \csc \phi + (\frac{f^2}{2}) \sin \phi \cos^2\phi]$$

It should be noted that the coefficients of the above two formulas display, respectively, the same product combinations in a_1 and m_1 as in a and m. Also, the bracketed quantities of each are a function of an arc length (ϕ_S or ϕ) and a spheroid parameter (e'^2 or f). However, whereas e'^2 or e'^4 is factorable from the bracketed terms of the first equation, $(f + f^2)$ is not factorable from the second equation without leaving other f terms still within the brackets.

The capability of factoring out the spheroid parameters from both formulas would be desirable from the practical and theoretical point of view. From the practical point of view, there would then be required a single geodetic table from which to interpolate the bracketed values as a function of ϕ_S or ϕ, independent of spheroid. (The formulas would also be somewhat simpler for electronic programming and computing.) From the theoretical point of view, it would provide additional insight and interest relative to the intrinsic mathematical properties and relationships of geodesics. This is in line with other relationships noted in [1], such as the sum of the numerical coefficients in each of the four power series adding to zero for certain series of terms.

An attempt to satisfactorily factor out the spheroid parameter from the last remaining power series (that of the Inverse Distance) was made in the last paragraph at the bottom half of page 88 of [1]. However, the following (which is mathematically equivalent to the second formula of the present paper) is intended to replace that paragraph in [1]:

2

$$\frac{S}{b_0} = \qquad\qquad [(1 + f + f^2)\phi]$$

$$+ (m) \qquad\qquad [- (\frac{f + f^2}{2})\phi]$$

$$+ (2a - m\cos\phi) \qquad [(\frac{f + f^2}{2})\sin\phi]$$

$$+ (m^2) \qquad\qquad [(\frac{f^2}{16})(\phi + \sin\phi\cos\phi)]$$

$$+ (2a - m\cos\phi)^2 \qquad [- (\frac{f^2}{8})\sin\phi\cos\phi]$$

$$+ (1 - m)(a - m\cos\phi)[- (\frac{f^2}{2})\phi^2\csc\phi]$$

The above rigorous non-iterative solution includes terms which give an accuracy of up to a few centimeters for geodetic lines several thousand miles long. (Formulas of one higher degree of accuracy are given in the original papers, [1] and [2].) Yet the bracketed terms of the above formula are so fewer and simpler than those of the Direct as well as the Inverse formula, as given respectively at the beginning of this paper, that even a single tabulation applicable to any and all spheroids would not be necessary after the flattening functions are factored out. A formula in corresponding type of coefficients, m and (2a - m cos φ), can be written for the (λ - L) + c expression (used to obtain geodetic azimuths and difference of longitudes) at the top of page 74 of [1].

REFERENCES

1.* Sodano, E. M. - "General Non-Iterative Solution of the Inverse and Direct Geodetic Problems," Bulletin Geodesique, 1965, No. 75, pp. 69-89.

2.* Sodano, E. M. - "A Rigorous Non-Iterative Procedure for Rapid Inverse Solution of Very Long Geodesics," Bulletin Geodesique, 1958, No. 47/48, pp. 13-25.

* Errata sheets, not previously published, are attached.

ERRATA SHEET

"General Non-Iterative Solution of the Inverse and Direct Geodetic Problems" by E. M. Sodano, Bulletin Geodesique, 1965, No. 75, pp. 69-39.

Page 69, first word of line before last of paragraph I should be "beyond."

Page 70, in third line from end of paragraph III, a decimal point should be before the 5 instead of the period after previous word.

Page 70, in line 7 of paragraph IV, the word should be "parameter" instead of its plural.

Page 75, in third line of Appendix B, place comma before S.

Page 76, in heading of lower table, the correct spelling is "and."

Page 89, in third line of Appendix G, the word should be "indication" instead of its plural.

- -

ERRATA SHEET

"A Rigorous Non-Iterative Procedure for Rapid Inverse Solution of Very Long Geodesics" by Emanuel M. Sodano, Bulletin Geodesique, 1958, No. 47/48, pp. 13-25.

Page 21, at end of tenth line from end of page, place "cotangent" in front of α_{1-2} and in front of α_{2-1}.